

A New Method for Accelerating Impossible Differential Cryptanalysis and its Application on LBlock[☆]

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ABSTRACT

Impossible differential cryptanalysis, an extension of the differential cryptanalysis, is one of the most efficient attacks against block ciphers. This cryptanalysis method has been applied to most of the block ciphers, and has shown significant results. Using structures, key schedule considerations, early abort, and pre-computation are some common methods to reduce complexities of this attack. In this paper, we present a new method for decreasing the time complexity of impossible differential cryptanalysis through breaking down the target key space into subspaces, and extending the results on subspaces to the main target key space. The main advantage of this method is that there is no need to consider the effects of changes in the values of independent key bits on each other. Using the 14-round impossible differential characteristic observed by Boura *et al.* at ASIACRYPT 2014, we implement this method on 23-round LBlock and demonstrate that it can reduce the time complexity of the previous attacks to $2^{71.8}$ 23-round encryptions using 2^{59} chosen plaintexts and 2^{73} blocks of memory.

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1 Introduction

1.1 Motivation, Contribution and Organization

Cryptanalysis methods that rely on eliminating the round's function key's effect by working on the differences, form a considerable part of attacks against block ciphers. Differential cryptanalysis [1] proposed by Biham *et al.* is the first attack of this cat-

egory. Impossible differential, higher order differential, truncated differential, and boomerang cryptanalyses are other attacks of this type.

Impossible differential cryptanalysis was introduced independently by Knudsen on DEAL block cipher [7] and Biham *et al.* on Skipjack block cipher [2]. It is one of the conventional cryptanalyses methods for block ciphers showing remarkable results. Distinguisher of impossible differential attack, impossible differential characteristic, is n -round of target algorithm with specific input and output differences holding with probability zero, implying that a pair with such input difference cannot lead to the specified output difference after n rounds. After extending the characteristic by adding some rounds to the plaintext and ciphertext sides of the distinguisher, we try to find the key val-

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ues leading chosen pairs of plaintexts to the attack’s distinguisher. Since these key values lead to the distinguisher holding with probability zero; they cannot be the correct key and should be eliminated from the target key space.

LBlock is a lightweight block cipher of generalized Feistel type represented in ACNS2011 [3]. The algorithm consists of 32 rounds with 64-bit block length, and 80-bit key length. In [3], the designers evaluated LBlock’s security against different attacks including impossible differential cryptanalysis. Using a 14-round impossible differential characteristic They presented the attack on 20 rounds of the algorithm. Using another 14-round characteristic, impossible differential cryptanalysis on 21 rounds of the algorithm is presented in [4]. This improvement was achieved by applying mentioned methods, such as key schedule consideration. The authors in [5] represented another impossible differential cryptanalysis of LBlock. They have separated the plaintext and ciphertext sides added to the distinguisher and searched each part independently. Using this method, they could reduce the complexity on 21 rounds of algorithm. Also, they have extended the attack to 22 rounds. Our new method can be considered as an extension of their work. By investigating the key schedule of the cipher in [9], the authors have extended impossible differential cryptanalysis to 23 rounds of the algorithm. In [10], the authors presented another impossible differential cryptanalysis on 23 rounds of the algorithm by using an approach that can help reducing the number of pairs used in the attack.

In conventional impossible differential attacks, the effects of changes in most of the target key bits are considered while the attackers are studying the possible values for some specific key bits, although lots of them are independent; and these considerations come to attacks with higher time complexities. The main idea of the new method is preventing this overload by separating the target key space into subspaces. We classify the target key bits into the groups and determine the values of key bits in each group independently. Then, we extend the achieved results to the whole target key space. This idea was first applied on 22-round LBlock in [21]; however, there was no efficient algorithm for extending the achieved results to the main target key space. In this paper, we generalize the method and propose an efficient algorithm for combining the results stored in distinct tables.

Using this improved method in parallel with previous ones, an improved attack on 23-round LBlock is presented in this paper. The results of this attack are compared with the previous works in Table 1. The outstanding advantages of this method are its generality that can be applied on different block ciphers

(especially those with weak diffusion layers including lightweight algorithms) and also applicability in parallel with other techniques proposed for improving impossible differential cryptanalysis. The rest of this pa-

Table 1. Comparison of attacks on LBlock

No. Rounds	Cryptanalysis method	Memory complexity	Data complexity	Time complexity	Ref.
18	Integral	2^{16}	2^{62}	2^{12}	[17]
20	Integral	2^{35}	$2^{63.6}$	$2^{39.6}$	[17]
20	Impossible Differential	2^{60}	2^{63}	$2^{72.7}$	[3]
21	Impossible Differential	2^{68}	$2^{62.5}$	$2^{73.7}$	[4]
21	Impossible Differential	2^{68}	2^{63}	$2^{69.5}$	[5]
22	Impossible Differential	$2^{72.67}$	2^{58}	$2^{79.28}$	[5]
22	Zero-Correlation Linear	2^{64}	2^{62}	$2^{71.27}$	[14]
22	Integral	2^{63}	2^{61}	2^{70}	[16]
22	Related Key Impossible Differential	not mentioned	2^{47}	2^{70}	[6]
23	Related Key Impossible Differential	$2^{61.4}$	$2^{61.4}$	$2^{78.3}$	[8]
23	Impossible Differential	not mentioned	2^{57}	$2^{77.4}$	[9]
23	Impossible Differential	$2^{74.6}$	2^{59}	$2^{75.36}$	[10, 12, 13]
23	Impossible Differential	2^{73}	2^{59}	$2^{71.8}$	This Paper
24	Key Difference Invariant Bias (Related-Key)	2^{61}	$2^{62.95}$	$2^{70.67}$	[19]
Full	Higher Order Key Partitioning	Negligible	2	$2^{78.338}$	[20]
Full	Biclique	Negligible	$2^{78.4}$	2^{52}	[18]

per is organized as follows: In Section 2, we introduce the impossible differential cryptanalysis and describe the new method. Prevalent notations for LBlock and brief description of it are presented in Section 3. In Section 4, we clarify the new method by applying it on LBlock. Finally, we conclude the paper in Section 5.

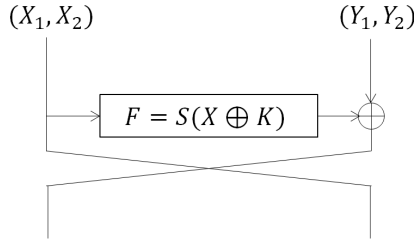


Figure 1. typical round function in algorithms of Feistel ciphers

2 Impossible Differential Cryptanalysis and the New Method

Consider the typical round function in block ciphers of Feistel type, consisted of add round key by XOR operation, and applying S-Boxes, as depicted in Figure 1. The terms used in this paper are as follows:

Definition 1 For a pair (X_1, Y_1) and (X_2, Y_2) , *Cancellation* occurs when $X_1 \neq X_2$ and $Y_1 \neq Y_2$ and $[F(X_1, K) \oplus F(X_2, K)] = [S(X_1 \oplus K) \oplus S(X_2 \oplus K)] = (Y_1 \oplus Y_2)$.

Cancellations on specific locations in the target algorithm are the *conditions* of the impossible differential attack. Key values satisfying the attack’s conditions result the attack’s characteristic from the chosen pairs. These values for the key are those we are looking for during the attack procedure.

Definition 2 For each cancellation, corresponding to one F-function, we form a group made up of the F-function’s subkey, and also the key bits of the other rounds which affect the input pair value or the desired differential output value of that F-function $(X_1, X_2$ and $Y_1 \oplus Y_2$ in definition 1). The number of key groups in the attack equals the number of cancellations.

Definition 3 *Involved-key-bits* is the set of key bits presented in groups.

Definition 4 *Information-key-bits* is a subset of involved-key-bits of the smallest size that can specify the involved-key-bits value uniquely.

Due to the redundancy of the key schedule, it is probable to have relations between different rounds’ subkeys in the set of involved-key-bits. As a result, size of involved-key-bits is less than or equal to the size of information-key-bits. There is a well-known property for invertible S-boxes that we apply it in our attack implementation:

Property 1 For F-function $F(X, K) = S(X \oplus K)$, where S is a given invertible S-Box, for an input pair (X_1, K) and (X_2, K) ; if we know the value of (X_1, X_2) , we can determine the values of K leading to a specified differential value for $[S(X_1 \oplus K) \oplus S(X_2 \oplus K)] = \Delta Y$.

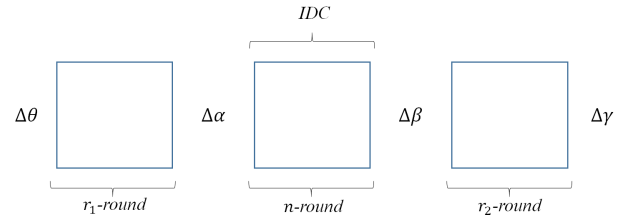


Figure 2. impossible differential cryptanalysis

2.1 Impossible Differential Cryptanalysis

The characteristic in impossible differential attack consists of n rounds of the target algorithm with $\Delta\alpha$ and $\Delta\beta$ as the differential input and output of this n -round characteristic, respectively (Figure 2). $\Delta\alpha$ (resp. $\Delta\beta$) contains m_1 (resp. m_2) non-zero difference sub-blocks (mostly $m_1 = m_2 = 1$). This characteristic states that a pair of texts with $\Delta\alpha$ as the differential input to the round i cannot lead to $\Delta\beta$ as the differential output of the round $i + n - 1$.

Adding r_1 rounds to the plaintext side and r_2 rounds to the ciphertext side of the characteristic, impossible differential attack can be implemented on $n + r_1 + r_2$ rounds of the algorithm, as Figure 2. After this extension, differential input (resp. output) of the $n + r_1 + r_2$ -round version of the algorithm equals $\Delta\theta$ (resp. $\Delta\gamma$) containing $l_1 + m_1$ (resp. $l_2 + m_2$) non-zero difference sub-blocks. The number of conditions in this attack is $l_1 + l_2$. In this way, the attack is based on finding key values that cancel l_1 (resp. l_2) number of these non-zero difference sub-blocks on specific locations from the plaintext side (resp. ciphertext side), leading to the attack’s characteristic. Since the impossible differential characteristic comes to a contradiction, none of the key values leading to this characteristic can be the correct key.

For finding the incorrect key values, we take some pairs with the specified differential forms in the plaintext and ciphertext sides; $\Delta\theta$ and $\Delta\gamma$, respectively. Then, we test the possible key values on them to find those which lead to the impossible differential characteristic, i.e. incorrect keys. The number of pairs needed for recovering the correct key depends on the number of cancellations that determine the number of condition-bits (the bits that there are conditions on them). For $l_1 + l_2$ cancellations at t -bit F-functions, the number of condition-bits equals $(l_1 + l_2)t$. If the number of information-key-bits in the attack is s , each pair can discard $2^s(2^{-(l_1+l_2)t})$ key values on average. So, if the attacker wants to eliminate all the incorrect keys, the number of required pairs, q , is determined by the following inequality:

$$2^s(1 - 2^{-(l_1+l_2)t})^q \leq 1$$

Time and memory complexities of impossible differential cryptanalysis are dependent on the attack

procedure. In [11] a lower bound for time complexity of impossible differential cryptanalysis is provided. The authors prove that an impossible differential attack, which gives 2^ε values for the L -bit involved keys, contains at least $(L - \varepsilon + \ln(2)) \times 2^L$ memory accesses.

In conventional impossible differential cryptanalysis, there are some common methods to reduce the attack's complexity; such as using structures, key schedule considerations, early abort, and pre-computation. Using structures leads attackers to choose texts of specific form in which there are constant values in some positions of the texts' block and the other positions can take all possible values. By this method, the chosen pairs from one structure have the desired differential form. Key schedule consideration is one of the most effective methods to reduce the complexity of attacks. The key schedule generates rounds' subkeys from one seed, main key, usually in an iterative form; and therefore, it contains some redundancy. In ideal key schedules, this redundancy is not exploitable; but for most of the algorithms, the attackers can find some applicable redundancy to reduce size of the target key space. In early abort technique, attackers discard the pairs or key values that cannot comply with some conditions as soon as possible. Here, pre-computation means performing the repetitive computations that are independent from the achieved results of the attack's steps and classifying them in tables before the attack starts. Then, attackers can refer to these tables and find the result of the intended computation when they need, instead of spending time on computing in the online phase of the attack.

2.2 The New Method

In impossible differential attack, the attackers are searching for the values of the information-key-bits satisfying the attack's conditions to discard them from the target key space. Not necessarily all the information-key-bits have effect on all the conditions. However, in conventional impossible differential cryptanalysis methods, the effects of changes in the values of all the information-key-bits are considered while attackers are studying a specific cancellation. Here, we present a new method which is based on reducing computational complexity by refusing to study the effects of independent key bits' changes on the attack's conditions.

In our attack scenario, for each one of $l_1 + l_2$ cancellations, we determine the effective key bits and put them in distinct groups. Then, for each group, we form a table consisted of flags corresponding to all the possible values of the determined key bits. For each chosen pair, we find the key values satisfying each cancellation and set the achieved values' flags in the

corresponding table. The combinations of key values that their flags are set from the tables lead the chosen pair to the attack's distinguisher and should be eliminated from the target key space.

According to property 1, we just need to know the input pair value of (X_1, X_2) and the desired output difference of the F-function $F = S(X \oplus K)$ for each cancellation to determine the key values of the function; and just the changes in the key bits that affect the input pair value or the desired differential output value of the F-function are needed to be considered, not the changes in all the involved-key-bits. Therefore, there are two types of key bits in each group:

Type-1: key bits of the F-function on which we have cancellation,

Type-2: key bits which affect the input pair value or the desired differential output value of this function

(Groups of key bits corresponding to cancellations on the first and last rounds do not contain key bits of type-2.)

For each group of key bits that contains key bits of type-2, we guess this type of key bits to determine the input pair value and the desired differential output value of the F-function. Then, we can determine the values of type-1 key bits in each group.

Impossible differential cryptanalysis by using the new method can be done within the following steps:

- (1) Choosing a proper n -round impossible differential characteristic,
- (2) Extending number of rounds to $n + r_1 + r_2$,
- (3) Determining the non-zero difference sub-blocks of the plaintext and ciphertext sides,
- (4) Specifying number of cancellations $l_1 + l_2$ and their locations,
- (5) Forming $l_1 + l_2$ key groups corresponding to $l_1 + l_2$ cancellations and determining each group's key bits,
- (6) Forming $l_1 + l_2$ tables corresponding to $l_1 + l_2$ key groups consisted of flags corresponding to the possible values of the determined key bits for each group,
- (7) Choosing pairs with the specified difference in the plaintext and ciphertext sides, and repeating the two following steps for each pair,
- (8) Determining key bits values of each groups and setting their corresponding values' flags in the corresponding table,
- (9) Eliminating the combinations of key bits values of tables that their flags are set from the target key space.

We clarify this method in Section 4 by applying it on 23-round LBlock. To the best of our knowledge,

impossible differential attack by this method comes to the best results in terms of data and time complexities, considering the cryptanalysis methods in single-key model excluding biclique attack.

3 Brief Description of LBlock

The used notation in this paper is summarized here:

- A : a bit string
- $A|B$: concatenation of strings A and B
- $A \lll B$: left rotation of A by j bits
- K_i^r : the i^{th} nibble of the r^{th} round key
- R^{r-1} : the right half of the r^{th} round's input
- L^{r-1} : the left half of the r^{th} round's input
- R_i^{r-1} : the i^{th} nibble of R^{r-1}
- L_i^{r-1} : the i^{th} nibble of L^{r-1}
- ΔR^{r-1} : the difference of two R^{r-1}
- ΔL^{r-1} : the difference of two L^{r-1}
- ΔR_i^{r-1} : the difference of two R_i^{r-1}
- ΔL_i^{r-1} : the difference of two L_i^{r-1}
- S_i^r : output of the i^{th} S-Box in the r^{th} round
- ΔS_i^r : the difference of two S_i^r

(Note that the nibbles are indexed beginning with zero subscript and ending with seven, and the zero-indexed nibble is the rightmost one)

LBlock is a lightweight block cipher with 64-bit block length and 80-bit key length [3]. Its structure is a generalization of a Feistel structure with 32 rounds. It can be implemented efficiently both in hardware and software. The iterative round function F consists of three basic functions: (1) add round key by XOR, (2) confusion function by S-Boxes, and (3) diffusion function by permutation and rotation. LBlock's algorithm uses 4-bit S-Boxes $S_i, i = 0, 1, \dots, 9$; where $S_i, i = 0, 1, \dots, 7$ are used in the round functions and S-Boxes S_8 and S_9 are used in the key schedule. Its round function is depicted in Figure 3.

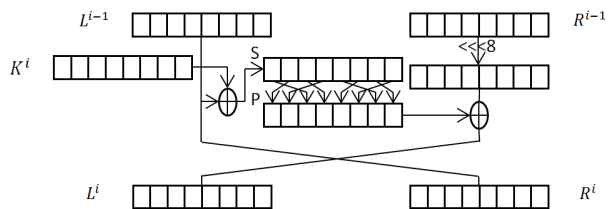


Figure 3. Round function of LBlock.

4 Improved Impossible Differential Cryptanalysis on LBlock

S-Boxes used in an algorithm are so effective in the security against cryptanalyses. After studying differential distribution tables of the S-Boxes of LBlock, $S_i, i = 0, 1, \dots, 7$ we found a property common among them:

Property 2 For the SBoxes used in the LBlock's algorithm, the probability of generating a specific output difference from an input difference is $2^{(-1.4)}$; and for each possible input pair and output difference, there are on average $2^{1.4}$ key value.

4.1 Impossible Differential characteristic

We use the 14-round impossible differential characteristic as our attack's distinguisher [10]. This characteristic is as follows (see Figure 5 in the Appendix):

$(0000, 0000, 0000, a000) \rightarrow (0000, 0b00, 0000, 0000)$
 Where $a, b \in \{0, 1\}^4 \setminus \{0\}^4$ are two non-zero nibbles. We add five rounds to the plaintext side and four rounds to the ciphertext side of the characteristic and propose our attack on 23-round LBlock (Figure 4).

4.2 Separating Target Key Space into Subspaces

There are eighteen cancellations during the attack; and therefore, we separate target key bits into eighteen groups. The groups $g_i, i = 1, 2, 3, \dots, 18$ and their corresponding conditions are demonstrated in Table 2. We find key values of each group, committing the corresponding cancellation, and discard the combinations of these keys values from the target key space.

4.3 Pre-computation

For $S_i, i = 0, 1, \dots, 7$ we form the tables $A_i, i = 0, 1, \dots, 7$ indexed by $(x_1, x_2, \Delta y)$. For all possible values of $(x_1, x_2, \Delta y)$, we find key values which result specific values for $\Delta y = [S_i(x_1 \oplus k) \oplus S_i(x_2 \oplus k)], i = 0, 1, \dots, 7$; and put them in the corresponding rows of $A_i, i = 0, 1, \dots, 7$. Also, for 18 groups of key bits, we form tables $U_i, i = 1, 2, 3, \dots, 18$ consisted of flags corresponding to the possible values for the key bits in each group.

4.4 Key Recovery

The key recovery procedure is organized as the following steps:

Step 1 (Choosing plaintexts). Take 2^n structures of the form $(b_1|a_1|b_2|a_2)|(b_3|b_4|b_5|a_3)|(b_6|b_7|b_8|b_9)|(b_{10}|b_{11}|b_{12}|a_4)$ where $a_i, i = 1, 2, 3, 4$, are 4-bit fixed constants, and each $b_i, i =$

Table 3. Details of step 3

Cancellation NO.	Corresponding S-Box	Diff. Input and Output	Equivalent Diff. Input and Output	Complexity (memory access)	No. of Survived Pairs
1	S_1	$(\Delta L_1^0, \Delta R_6^0)$	$(\Delta L_1^0, \Delta R_6^0)$	2^{n+63}	$2^{n+61.6}$
2	S_3	$(\Delta L_3^0, \Delta R_7^0)$	$(\Delta L_3^0, \Delta R_7^0)$	$2^{n+61.6}$	$2^{n+60.2}$
3	S_2	$(\Delta L_2^0, \Delta R_1^0)$	$(\Delta L_2^0, \Delta R_1^0)$	$2^{n+60.2}$	$2^{n+58.8}$
4	S_5	$(\Delta L_5^0, \Delta R_2^0)$	$(\Delta L_5^0, \Delta R_2^0)$	$2^{n+58.8}$	$2^{n+57.4}$
6	S_7	$(\Delta L_7^1, \Delta R_3^1)$	$(\Delta L_5^0, \Delta R_3^0)$	$2^{n+57.4}$	2^{n+56}
7	S_6	$(\Delta L_6^1, \Delta R_5^1)$	$(\Delta L_4^0, \Delta R_5^0)$	2^{n+56}	$2^{n+54.6}$
8	S_1	$(\Delta L_1^2, \Delta R_6^2)$	$(\Delta L_7^0, \Delta R_4^0)$	$2^{n+54.6}$	$2^{n+53.2}$
9	S_3	$(\Delta L_3^2, \Delta R_7^2)$	$(\Delta L_1^0, \Delta R_5^0)$	$2^{n+53.2}$	$2^{n+51.8}$
12	S_0	$(\Delta R_0^{23}, \Delta L_2^{23})$	$(\Delta R_0^{23}, \Delta L_2^{23})$	$2^{n+51.8}$	$2^{n+50.4}$
13	S_5	$(\Delta R_5^{23}, \Delta L_4^{23})$	$(\Delta R_5^{23}, \Delta L_4^{23})$	$2^{n+50.4}$	2^{n+49}
14	S_6	$(\Delta R_6^{23}, \Delta L_7^{23})$	$(\Delta R_6^{23}, \Delta L_7^{23})$	2^{n+49}	$2^{n+47.6}$
15	S_1	$(\Delta L_1^{21}, \Delta L_0^{22})$	$(\Delta L_3^{23}, \Delta R_0^{23})$	$2^{n+47.6}$	$2^{n+46.2}$
16	S_4	$(\Delta L_4^{21}, \Delta L_6^{22})$	$(\Delta L_6^{23}, \Delta R_6^{23})$	$2^{n+46.2}$	$2^{n+44.8}$
17	S_3	$(\Delta L_3^{20}, \Delta L_1^{21})$	$(\Delta L_5^{23}, \Delta R_3^{23})$	$2^{n+44.8}$	$2^{n+43.4}$
18	S_2	$(\Delta L_2^{19}, \Delta L_3^{20})$	$(\Delta L_6^{23}, \Delta R_5^{23})$	$2^{n+43.4}$	2^{n+42}

Table 4. Differential input and output of the S-Boxes corresponding to the cancellations 5, 10, and 11.

Cancellation NO.	Corresponding S-Box	Differential Input and Output	Equivalent Differential Input and Output
5	S_5	$(\Delta L_5^1, \Delta R_2^1)$	$(\Delta L_5^1, \Delta L_2^0)$
10	S_7	$(\Delta L_7^3, \Delta R_3^3)$	$(\Delta L_5^1, \Delta L_1^0)$
11	S_3	$(\Delta L_3^4, \Delta R_7^4)$	$(\Delta L_7^0, \Delta L_5^1)$

since either the differential input or output of S-Boxes corresponding to cancellations 5, 10, and 11 is ΔL_5^1 (Table 4) which is dependent to the subkey K_7^1 , we put off verification for these cancellations until we guess the values for K_7^1 in this step. For 2^4 possible values of K_7^1 , calculate 2^4 values of (L_5^1, L_5^1) . Check the differential distribution table of S-Boxes S_5, S_7 and S_3 for cancellations 5, 10 and 11, respectively; and discard the values of K_7^1 that their corresponding values in the differential distribution tables are zero. According to property 2, the probability of surviving from these filtrations is $(2^{-1.4})^3 = 2^{-4.2}$; and $2^4 \times 2^{-4.2} = 2^{-0.2}$ values of K_7^1 pass this step. This step contains $(22)^4$ F-function and $2^4 + 2^{2.6} + 2^{1.2}$ memory accesses.

Step 6 (Cancellation 5). For $2^{-0.2}$ possible values of K_7^1 , calculate $2^{-0.2}$ values of (L_5^1, L_5^1) . Refer to the $(L_5^1, L_5^1, \Delta R_2^1)^{th}$ row of table A_5 to find the values of K_5^2 . Set the flags of the achieved values for $(K_7^1|K_5^2)$ in table U_5 . This step contains $2 \times 2^{-0.2}$ F-function

and $(2^{-0.2} + 2^{1.2})$ memory accesses.

Step 7 (Cancellation 6). For 2^4 possible values of K_6^1 , calculate 2^4 values of (L_7^1, L_7^1) . Refer to the $(L_7^1, L_7^1, \Delta R_3^1)^{th}$ row of table A_7 to find the values of K_7^2 . Set the flags of the achieved values for $(K_6^1|K_7^2)$ in table U_6 . This step contains 2×2^4 F-function and $(2^4 + 2^{5.4})$ memory accesses.

Step 8 (Cancellation 7). For 2^4 possible values of K_4^1 , calculate 2^4 values of (L_6^1, L_6^1) . Refer to the $(L_6^1, L_6^1, \Delta R_5^1)^{th}$ row of table A_6 to find the values of K_6^2 . Set the flags of the achieved values for $(K_4^1|K_6^2)$ in table U_7 . This step contains 2×2^4 F-function and $(2^4 + 2^{5.4})$ memory accesses.

Step 9 (Cancellation 8). For $2^{1.4}$ possible values of K_2^1 from table U_3 , and 2^4 possible values of K_3^2 , calculate $2^{5.4}$ values of (L_1^2, L_1^2) . Refer to the $(L_1^2, L_1^2, \Delta R_6^2)^{th}$ row of table A_1 to find the values of K_1^3 . Set the flags of the achieved values for $(K_2^1|K_3^2|K_1^3)$ in table U_8 . This step contains $2 \times 2^{5.4}$ F-function and $(2^{5.4} + 2^{6.8})$ memory accesses.

Step 10 (Cancellation 9). For 2^8 possible values of $K_0^1|K_2^2$, calculate 2^8 values of (L_3^2, L_3^2) . Refer to the $(L_3^2, L_3^2, \Delta R_7^2)^{th}$ row of table A_3 to find the values of K_3^3 . Set the flags of the achieved values for $K_0^1|K_2^2|K_3^3$ in table U_9 . This step contains 2×2^8 F-function and $(2^8 + 2^{9.4})$ memory accesses.

Step 11 (Cancellation 10). For $2^{-0.2}$ possible values of K_7^1 from step 5, $2^{1.4}$ values of K_5^1 in table U_4 , and

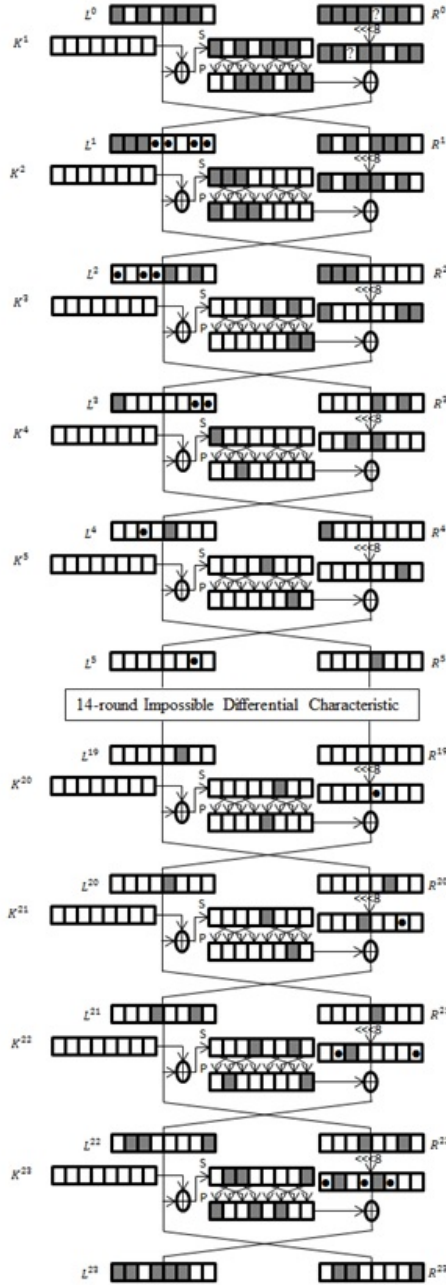


Figure 4. Round function of LBlock. White boxes represent 4-bit zero difference; black boxes present 4-bit non-zero differences; boxes containing question mark present 4-bit either zero or non-zero differences; and boxes containing black circle show the locations of the cancellations.

2^8 possible values of $K_4^2|K_6^3$, calculate $2^{9.2}$ values of (L_7^3, L_7^3) . Refer to the $(L_7^3, L_7^3, \Delta R_1^0)^{th}$ row of table A_7 to find the values of K_7^4 . Set the flags of the achieved values for $K_7^1|K_5^1|K_4^2|K_6^3|K_7^4$ in table U_{10} . This step contains $2 \times 2^{9.2}$ F-function and $(2^{9.2} + 2^{10.6})$ memory accesses.

Step 12 (Cancellation 11). According to the key schedule, the most significant bit of K_2^4 is equal to the least significant bit of K_1^1 . Also, the two least

significant bits of K_0^3 are equal to the two most significant bits of K_5^1 . Utilizing these relations among subkeys bits, for $2^{1.4}$ values of K_3^1 in table U_2 , $2^{1.4}$ values of K_2^1 in table U_3 , $2^{1.4}$ values of K_1^1 in table U_1 , $2^{3.35}$ values of K_2^2 using table U_1 , $2^{2.7}$ values of K_0^2 using table U_4 , and 2^8 possible values of $K_3^2|K_1^2$, calculate $2^{18.25}$ values of (L_3^4, L_3^4) . For $2^{-0.2}$ possible values of K_7^1 from step 5, calculate $2^{-0.2}$ possible values of ΔR_7^4 . For $2^{18.25} \times 2^{-0.2} = 2^{18.05}$ possible values of $(L_3^4, L_3^4, \Delta R_7^4)$, refer to the $(L_3^4, L_3^4, \Delta R_7^4)^{th}$ row of table A_3 to find the values of K_3^5 . Set the flags of the achieved values for $K_7^1|K_3^1|K_2^1|K_1^1|K_3^2|K_1^2|K_0^2|K_2^2|K_3^5$ in table U_{11} . This step contains $2 \times (2^{18.25} + 2^{-0.2})$ F-function and $(2^{18.05} + 2^{19.45})$ memory accesses.

Step 13 (Cancellation 15). For 2^4 possible values of K_2^{23} , calculate 2^4 values of (L_1^{21}, L_1^{21}) . Refer to the $(L_1^{21}, L_1^{21}, \Delta L_0^{22})^{th}$ row of table A_1 to find the values of K_1^{22} . Set the flags of the achieved values for $(K_2^{23}|K_1^{22})$ in table U_{15} . This step contains 2×2^4 F-function and $(2^4 + 2^{5.4})$ memory accesses.

Step 14 (Cancellation 16). For 2^4 possible values of K_4^{23} , calculate 2^4 values of (L_4^{21}, L_4^{21}) . Refer to the $(L_4^{21}, L_4^{21}, \Delta L_6^{22})^{th}$ row of table A_4 to find the values of K_4^{22} . Set the flags of the achieved values for $(K_4^{23}|K_4^{22})$ in table U_{16} . This step contains 2×2^4 F-function and $(2^4 + 2^{5.4})$ memory accesses.

Step 15 (Cancellation 17). For 2^8 possible values of $K_3^{23}|K_7^{22}$, calculate 2^8 values of (L_3^{20}, L_3^{20}) . Refer to the $(L_3^{20}, L_3^{20}, \Delta L_1^{21})^{th}$ row of table A_3 to find the values of K_3^{21} . Set the flags of the achieved values for $K_3^{23}|K_7^{22}|K_3^{21}$ in table U_{17} . This step contains 2×2^8 F-function and $(2^8 + 2^{9.4})$ memory accesses.

Step 16 (Cancellation 18). For 2^{16} possible values of $K_1^{23}|K_4^{23}|K_6^{22}|K_5^{21}$, calculate 2^{16} values of (L_2^{19}, L_2^{19}) . Refer to the $(L_2^{19}, L_2^{19}, \Delta L_3^{20})^{th}$ row of table A_2 to find the values of K_2^{20} . Set the flags of the achieved values for $K_1^{23}|K_4^{23}|K_6^{22}|K_5^{21}|K_2^{20}$ in table U_{18} . This step contains 2×2^{16} F-function and $(2^{16} + 2^{17.4})$ memory accesses.

Now, we have the key values for 18 cancellations in tables; and the combinations of these key values are the incorrect keys that should be eliminated from the candidate key space.

Step 17 (Combining the results). In this step, we combine the achieved results stored in tables U_i , $i = 1, 2, \dots, 17, 18$ to find the incorrect key values and eliminate them. Algorithm 1 shows an efficient method for this purpose. In this algorithm, we use the redundancy of the key schedule to reduce the number of times needed to access the tables. In this way, for a chosen combination of subkeys values from tables $U_1 U_k$, $k = 1, 2, \dots, 17$ that their corresponding flags

are set, we just check the flags of the subkeys values in table $U_{(k+1)}$ that they comply with the chosen combination according to the redundancy of the key schedule. If their flags are set, they will come to a new combination for being discarded from the target key space.

For instance, for the values of the subkey in $g_1 = K_1^1$, first we chose those that their flags are set in U_1 . Then, for each one of the chosen subkey values, we just check the flags of the subkey values of $g_2 - g_1g_2$ in table U_2 ; where $g_i g_j$ presents the key bits common between g_i and g_j . In this way, if g_1 and g_2 have any bit in common, we just check the flags of the values in U_2 that they comply with the chosen value from U_1 in terms of the chosen value for the common bits from U_1 . Generalizing this method for accessing the

Algorithm 1 Eliminating the combinations of the achieved results in tables $U_i, i = 1, \dots, 18$ from target key space.

```

for  $2^{(n+42)}$  pairs do
  for the subkey values in  $g_1$  do
    if the flag is set in  $U_1$  then
      for the the subkey values in  $g_2 - g_1g_2$  do
        if the flag is set in  $U_2$  then
          for the the subkey values in  $g_3 - g_1g_3 - g_2g_3$ 
            do
              if the flag is set in  $U_3$  then
                :
                for the the subkey values in  $g_{18} - g_1g_{18} -$ 
                 $g_2g_{18} - g_3g_{18} - \dots - g_{17}g_{18}$  do
                  if the flag is set in  $U_{18}$  then
                    eliminate the 73-bit key value from
                    the target key space
                  end if
                end for
              end if
            end for
          end if
        end for
      end if
    end for
  end for

```

table $U_{(k+1)}$ for a chosen combination from tables $U_1 \dots U_k, k = 1, 2, \dots, 17$, we minimize the time complexity of this step by using the redundancy of the key schedule. In other word, we do not access tables $U_i, i = 1, 2, 3, \dots, 18$ to check the flags of the subkeys values that cannot be correct according to the key schedule; and reduce the time complexity of this step in this way. Considering the redundancy of the key schedule, there are 2^{73} candidates for the key value in tables $U_i, i = 1, \dots, 18$, for being eliminated from target key space [10]. Among these 2^{73} values, we just pick those that their corresponding flags are set in tables $U_i, i = 1, \dots, 18$. The probability of this condition being true for each key value is $2^{(15(-2.6)+3(-4))} =$

$2^{(-51)}$, where $2^{(-2.6)}$ is the probability for each one of the 15 conditions 1, 2, \dots , 18, excluding conditions 5, 10 and 11, and $2^{(-4)}$ is the probability for each one of the conditions 5, 10, and 11 holding true. So, the time complexity of this step is $2^{(n+42)} \times 2^{73} \times 2^{(-51)} \times 19 \cong 2^{(n+68.2)}$ memory accesses (18 memory accesses for tables $U_i, i = 1, \dots, 18$, and one memory access for eliminating the incorrect key value).

4.5 Complexity

Since the attack contains $18 \times 4 = 72$ bits sieving, condition bits, the probability of being chosen for each key value after studying each pair is 2^{-72} . So, the probability of not being chosen for each key value after studying each pair is $(1 - 2^{-72})$. The number of information-key-bits is 73 [10], and after studying m pairs, $N = 2^{73}(1 - 2^{-72})^m$ key values remain in the target key space. Putting $m = 2^{74}$, the number of remained key values is $N = 2^{67.2}$; and we find the correct key among the remained key by exhaustive search. From $m = 2^{n+63} = 2^{74}$, n is 11; and the data complexity of attack is $2^{n+48} = 2^{59}$ chosen plaintexts.

Since we eliminate the incorrect key values from the target key space, we need a memory of size 2^{73} to keep the whole possible values for the target key bits and eliminate the incorrect values among them. So, the memory complexity of the attack is 2^{73} ; and sizes of the other used memories are negligible.

The time complexities of the attack's steps are indicated in Table 5. Dominant parts of time complexity refer to steps 3 and 17. Putting $n = 11$, the time complexity of attack is equivalent to $(2^{11+65} + 2^{11+68.2}) \times (\frac{1}{8} \times \frac{1}{23}) = 2^{71.8}$ 23-round encryption.

5 Conclusion

In this paper, we presented a new method for impossible differential cryptanalysis, based on separating target key space into subspaces. To this aim, we search the subspaces separately; then, we extend the combinations of the results to the whole target key space. We applied this method on 23 rounds of LBlock. The presented cryptanalysis requires 2^{59} chosen plaintexts and the time complexity is equivalent to $2^{71.8}$ encryptions, which is the best achieved results to the best of our knowledge. The achieved improvement is independent to those that are proposed so far; and studying their superposition property may lead to more efficient impossible differential attacks. Moreover, despite the fact that the proposed method does not necessarily lead to the most efficient impossible differential attack for all block cipher algorithms, its consideration is important while evaluating the security of an algorithm against this cryptanalysis method.

Table 5. Impossible differential cryptanalysis of 23-round LBlock

Step No.	Target Group	Target Key Bits	No. of Survived Key Values per Pair	Time Complexity		
2	None			2^{n+48} 23-round encryption		
3	None			2^{n+65} memory accesses		
4	1	K_1^1	$2^{1.4}$	$2^{n+42} \times 7(1 + 2^{1.4})$ memory accesses		
	2	K_3^1	$2^{1.4}$			
	3	K_2^1	$2^{1.4}$			
	4	K_5^1	$2^{1.4}$			
	12	K_0^{23}	$2^{1.4}$			
	13	K_5^{23}	$2^{1.4}$			
5	5,10,11	K_7^1	$2^{-0.2}$	$2^{n+42} \times 2 \times 2^4$ F-function		
				$2^{n+42} \times (2^4 + 2^{2.6} + 2^{1.2})$ memory accesses		
6	5	K_7^1	$2^{-0.2}$	$2^{n+42} \times 2 \times 2^{-0.2}$ F-function		
			K_5^2	$2^{1.4}$	$2^{n+42} \times (2^{-0.2} + 2^{1.2})$ memory accesses	
7	6	K_6^1	2^4	$2^{n+42} \times 2 \times 2^4$ F-function		
			K_7^2	$2^{1.4}$	$2^{n+42} \times (2^4 + 2^{5.4})$ memory accesses	
8	7	K_4^1	2^4	$2^{n+42} \times 2 \times 2^4$ F-function		
			K_6^2	$2^{1.4}$	$2^{n+42} \times (2^4 + 2^{5.4})$ memory accesses	
9	8	K_2^1	$2^{1.4}$	$2^{n+42} \times 2 \times 2^{5.4}$ F-function		
			K_3^2	2^4	$2^{n+42} \times (2^{5.4} + 2^{6.8})$ memory accesses	
			K_1^3	$2^{1.4}$		
10	9	K_0^1	2^4	$2^{n+42} \times 2 \times 2^8$ F-function		
			K_2^2	2^4	$2^{n+42} \times (2^8 + 2^{9.4})$ memory accesses	
			K_3^3	$2^{-0.2}$		
11	10	K_7^1	$2^{1.4}$	$2^{n+42} \times 2 \times 2^{9.2}$ F-function		
			K_5^1		2^4	
			K_4^2		2^4	
			K_6^3		2^4	$2^{n+42} \times (2^{9.2} + 2^{10.6})$ memory accesses
			K_7^4		$2^{1.4}$	
12	11	K_7^1	$2^{-0.2}$	$2^{n+42} \times 2 \times 2^{18.25}$ F-function		
			K_3^1		$2^{1.4}$	
			K_2^1		$2^{1.4}$	
			K_1^1		$2^{1.4}$	
			K_3^2		2^4	$2^{n+42} \times (2^{18.05} + 2^{19.45})$ memory accesses
			K_1^2		2^4	
			K_0^3		2^4	
K_2^4	2^4					
13	15	K_2^{23}	2^4	$2^{n+42} \times 2 \times 2^4$ F-function		
			K_1^{22}	$2^{1.4}$	$2^{n+42} \times (2^4 + 2^{5.4})$ memory accesses	
14	16	K_4^{23}	2^4	$2^{n+42} \times 2 \times 2^4$ F-function		
			K_4^{22}	$2^{1.4}$	$2^{n+42} \times (2^4 + 2^{5.4})$ memory accesses	
15	17	K_3^{23}	2^4	$2^{n+42} \times 2 \times 2^8$ F-function		
			K_7^{22}	2^4	$2^{n+42} \times (2^8 + 2^{9.4})$ memory accesses	
16	18	K_3^{21}	$2^{1.4}$	$2^{n+42} \times 2 \times 2^{16}$ F-function		
			K_1^{23}		2^4	
			K_4^{23}		2^4	
			K_6^{22}		2^4	
			K_5^{21}		2^4	$2^{n+42} \times (2^{16} + 2^{17.4})$ memory accesses
17	None	K_2^{20}	$2^{1.4}$	$2^{n+42} \times 2^{73} \times 2^{-51} \times 19 = 2^{n+68.2}$		

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Appendix

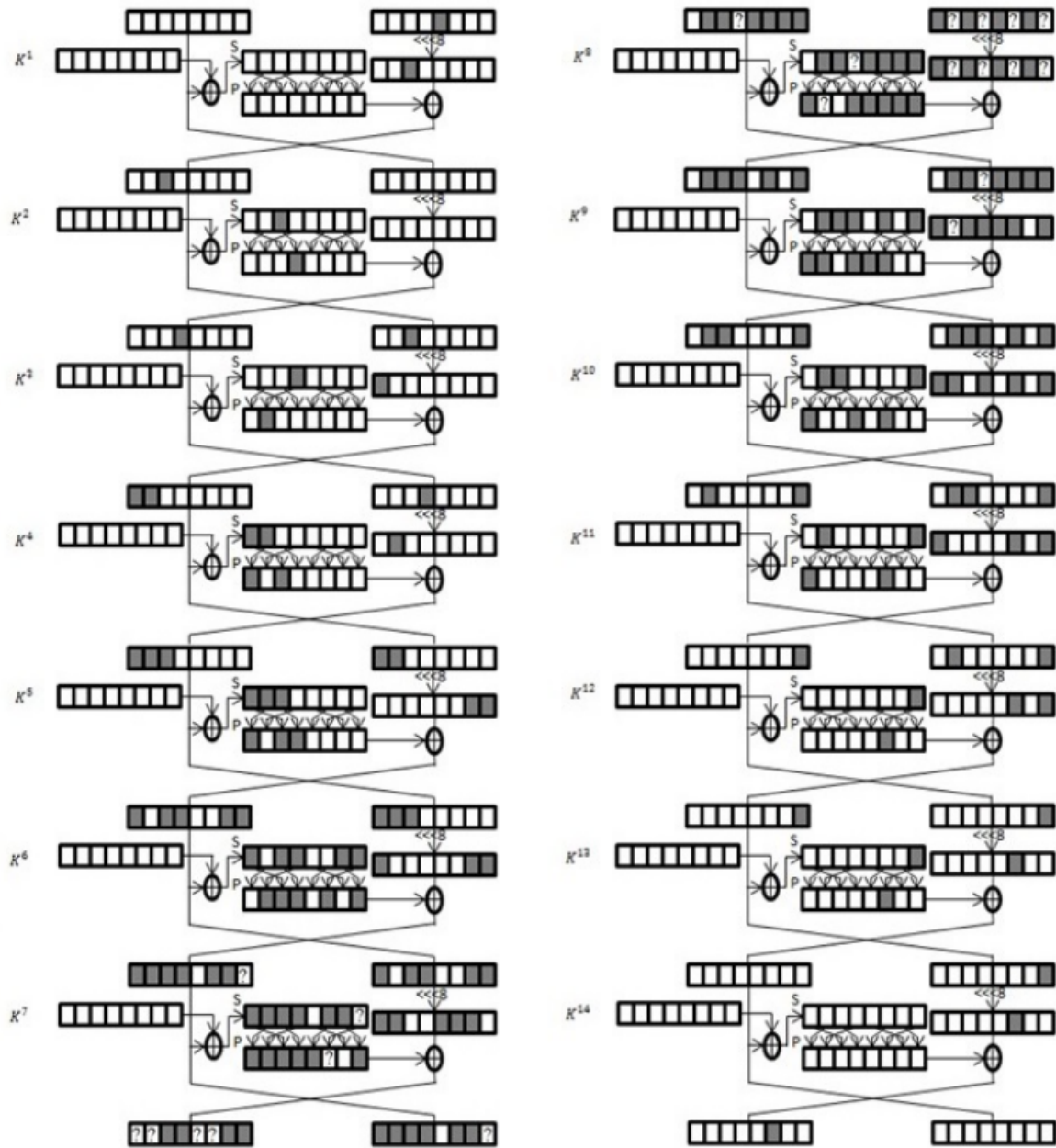


Figure 5. 14-round impossible differential characteristic. White boxes represent 4-bit zero difference; black boxes represent 4-bit non-zero differences; and boxes containing question mark represent 4-bit either zero or non-zero differences.